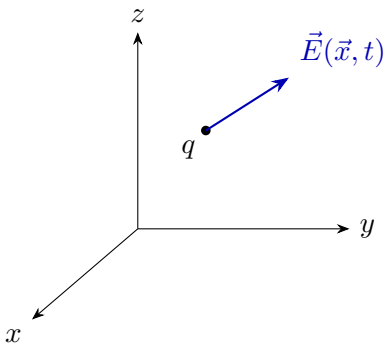
Chapter 1

Electric Fields & Capacitance

1.1 Electric field

The **electric field** $\vec{E}(\vec{x}, t)$ maps a point $\vec{x}$ and a time t to a vector $\vec{E}$.



Physically: $\vec{E}$ tells us how a test charge will move when placed at $\vec{x}$. The force on a test particle of charge q is

$$\boxed{\vec{F} = q\,\vec{E}} \quad \Longleftrightarrow \quad \vec{E} = \frac{\vec{F}}{q}. \tag{1.1}$$

Electric charge

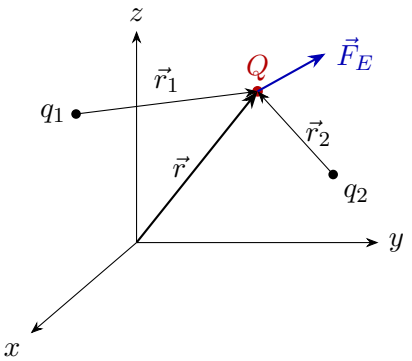
Electric charge plays two roles:

1. it is the propensity for *producing* an electric field;
2. it determines the force, $\vec{F}$ in *response* to an applied electric field. From (1.1), $q = |\vec{F}|/|\vec{E}|$.

Charges, at rest or in motion, are the sources of *all* electromagnetic fields.

1.2 Coulomb's law

Coulomb's law gives the electric force $\vec{F}_E$ that *static* point charges (i.e. in vacuum, at rest) exert on other charges.



$$\boxed{\vec{F}_E(\vec{r}) = \frac{Q}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i^2}\,\hat{r}_i} \tag{1.2}$$

where

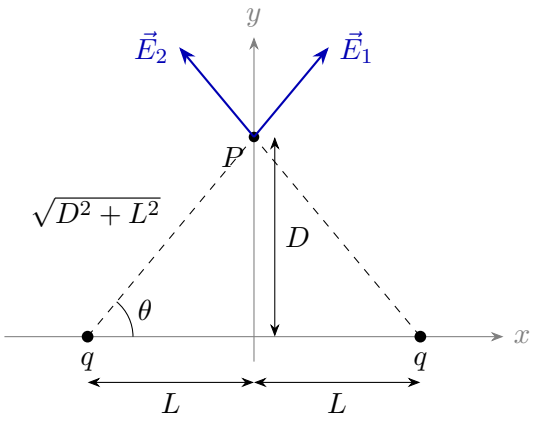
- Q is the charge of the particle experiencing $\vec{F}_E$;
- $\vec{r}_i$ is the separation vector from charge q_i to Q, with length $r_i = |\vec{r}_i|$ and unit vector $\hat{r}_i = \dfrac{\vec{r}_i}{|\vec{r}_i|}$;
- ε_0 is the permittivity of free space, $\varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$.

Combining (1.1) and (1.2) gives the electric field produced by N charged particles:

$$\boxed{\vec{E}(\vec{r}) = \frac{\vec{F}_E}{Q} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\,\hat{r}_i}$$

Example: two equal charges

Two charges q sit on the x-axis at $x = \pm L$. Find the field at point P, a height D above their midpoint.



By superposition, $\vec{E}(P) = \vec{E}_1 + \vec{E}_2 = (E_x^1 + E_x^2)\,\hat{\imath} + (E_y^1 + E_y^2)\,\hat{\jmath}$. By symmetry $E_x^2 = -E_x^1$ and $E_y^2 = E_y^1$, so

$$\vec{E}(P) = 2E_y^1\,\hat{\jmath}.$$

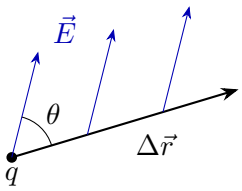
With $\sin\theta = D/\sqrt{D^2+L^2}$ and $r_1^2 = D^2 + L^2$,

$$E_y^1 = \frac{q}{4\pi\varepsilon_0}\frac{\sin\theta}{r_1^2} = \frac{q}{4\pi\varepsilon_0}\frac{D}{(D^2+L^2)^{3/2}},$$

and therefore

$$\boxed{\vec{E}(P) = \frac{q}{2\pi\varepsilon_0}\frac{D}{(D^2+L^2)^{3/2}}\,\hat{\jmath}}$$

1.3 Voltage



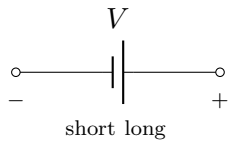
$|\Delta\vec{r}| = d$

Work done by the electric force on q as it moves through $\Delta\vec{r}$ in a uniform field:

$$W_E = \vec{F}_E \cdot \Delta\vec{r} = q\,E\,d\cos\theta.$$

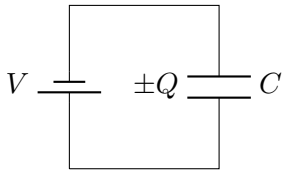
The **electric potential** difference, a.k.a. "voltage", is the (negative of the) work done per unit charge:

$$\boxed{\Delta V = -\frac{W_E}{q} = -E\,d\cos\theta} \tag{1.3}$$



A **battery** is a device that maintains a potential difference V between two conducting terminals. Its circuit symbol has two parallel lines of unequal length: the *longer* line is the positive terminal (higher potential) and the *shorter* line is the negative terminal (lower potential), so $V = V_+ - V_-$. We'll use this symbol for any constant voltage source.

1.4 Capacitors



A capacitor is made of two conducting plates across which a voltage V can be applied. One plate acquires charge $+Q$, the other $-Q$.

Capacitance: $\boxed{C = \dfrac{Q}{V} = \dfrac{\text{charge on plate}}{\text{voltage}}}$

Units: farad, F = C/V.

Parallel-plate capacitor

For a parallel-plate capacitor with plate area A and separation d, the field between the plates is uniform:

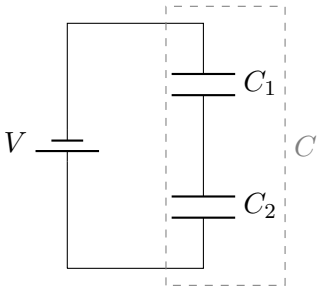
$$E = \frac{Q}{\varepsilon_0 A}.$$

Using (1.3) (magnitude, with $\theta = 0$), $V = Ed$, so

$$C_\parallel = \frac{Q}{V} = \frac{Q}{Ed} = \frac{Q}{\frac{Q}{\varepsilon_0 A}\,d} \quad\Longrightarrow\quad \boxed{C_\parallel = \frac{\varepsilon_0 A}{d}}$$

Note that Q cancels: capacitance depends only on geometry (a general result).

Capacitors in series



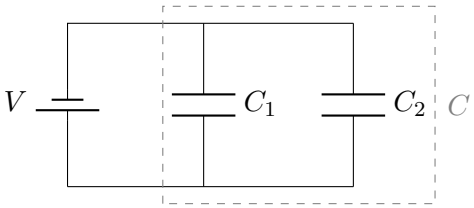
Let C be the effective capacitance of the combination.

- Conservation of energy (Kirchhoff's voltage law): $V_1 + V_2 = V$.
- Conservation of charge: the inner plates carry equal and opposite charge, $Q_1^+ = -Q_2^-$, so both capacitors hold the same Q.

$$C_1 = \frac{Q}{V_1}, \qquad C_2 = \frac{Q}{V_2} \qquad\Longrightarrow\qquad C = \frac{Q}{V} = \frac{Q}{\frac{Q}{C_1} + \frac{Q}{C_2}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Capacitors in parallel



Here the charges add and the voltages are equal:

$$Q = Q_1 + Q_2, \qquad V_1 = V_2 = V.$$

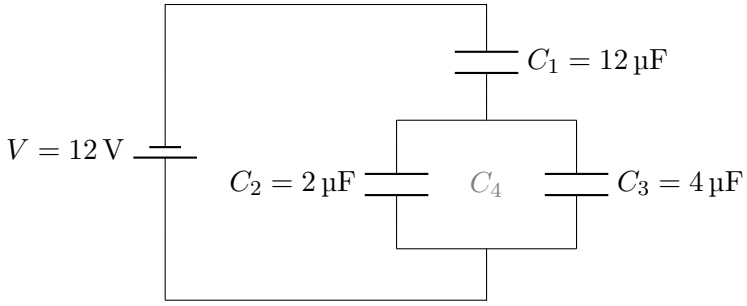
So

$$C = \frac{Q}{V} = \frac{Q_1 + Q_2}{V} = C_1 + C_2,$$

$$\boxed{C = C_1 + C_2}$$

Example

Find the effective capacitance of the circuit below.



C_2 and C_3 are in parallel, so they combine to $C_4 = C_2 + C_3 = 6\,\mu\text{F}$. C_4 is then in series with C_1:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_4} \implies C = \frac{C_1 C_4}{C_1 + C_4} = \frac{12 \cdot 6}{12 + 6}\,\mu\text{F} \qquad \implies \qquad \boxed{C = 4\,\mu\text{F}}$$

Problems

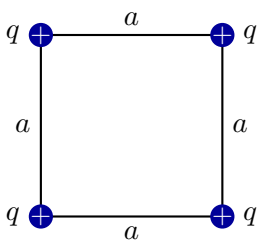
Problems are taken from the sources listed under Sources and Attribution. Numbers refer to the end-of-chapter problems; figures have been redrawn.

OpenStax University Physics Vol. 2, Chapter 5: Electric Charges and Fields

5.53 Point charges $q_1 = 50\,\mu\text{C}$ and $q_2 = -25\,\mu\text{C}$ are placed 1.0 m apart. What is the force on a third charge $q_3 = 20\,\mu\text{C}$ placed midway between q_1 and q_2?

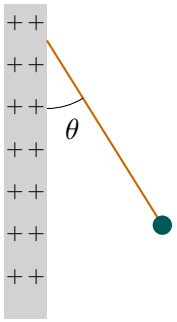
[Answer →]

5.63 What is the force on the charge q at the lower-right-hand corner of the square shown here?



[Answer →]

5.95 Shown is a small sphere of mass 0.25 g that carries a charge of 9.0×10^{-10} C. The sphere is attached to one end of a very thin silk string 5.0 cm long. The other end of the string is attached to a large vertical conducting plate that has a charge density of 30×10^{-6} C/m^2. What is the angle that the string makes with the vertical?



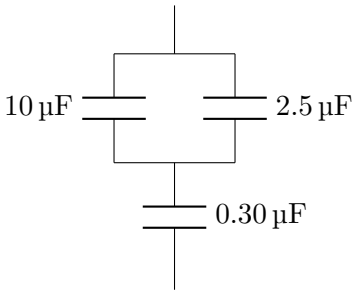
[Answer →]

OpenStax University Physics Vol. 2, Chapter 8: Capacitance

8.27 A set of parallel plates has a capacitance of 5.0 µF. How much charge must be added to the plates to increase the potential difference between them by 100 V?

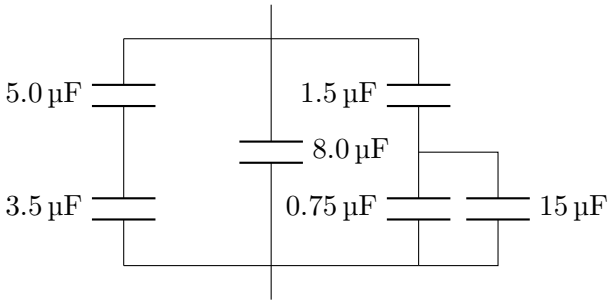
[Answer →]

8.33 Find the total capacitance of this combination of series and parallel capacitors.



[Answer →]

8.37 Find the net capacitance of the combination of series and parallel capacitors shown.



[Answer →]

Answers

Chapter 1: Electric Fields & Capacitance

5.53 $F = 53.94\,\mathrm{N}$ [← Problem]

5.63 $\vec{F}_{\mathrm{net}} = \left[k\frac{q^2}{a^2} + k\frac{q^2}{2a^2}\frac{1}{\sqrt{2}}\right]\hat{\imath} - \left[k\frac{q^2}{a^2} + k\frac{q^2}{2a^2}\frac{1}{\sqrt{2}}\right]\hat{\jmath}$ [← Problem]

5.95 $\theta = 32.0°$ [← Problem]

8.27 $500\,\mu\mathrm{C}$ [← Problem]

8.33 $0.29\,\mu\mathrm{F}$ [← Problem]

8.37 $11.4\,\mu\mathrm{F}$ [← Problem]

Sources and Attribution

The end-of-chapter problems in these notes are taken from the following openly licensed textbooks. Problem numbers in the text (for example 5.53) are the numbers of the end-of-chapter problems in the book named in the problems section.

- OpenStax, *University Physics Volume 2*, by Samuel J. Ling, Jeff Sanny and William Moebs. Licensed under CC BY 4.0. https://openstax.org/details/books/university-physics-volume-2
- OpenStax, *University Physics Volume 1*, by Samuel J. Ling, Jeff Sanny and William Moebs. Licensed under CC BY 4.0. https://openstax.org/details/books/university-physics-volume-1
- OpenStax, *Calculus Volume 3*, by Gilbert Strang, Edwin "Jed" Herman et al. Licensed under CC BY 4.0. https://openstax.org/details/books/calculus-volume-3

The signal-processing chapter also draws on the following book, which is not openly licensed and is cited rather than reproduced: Marina Bosi and Richard E. Goldberg, *Introduction to Digital Audio Coding and Standards*, Kluwer Academic Publishers, 2003. The approximation of the threshold in quiet used there is taken from that book; the plot of it is our own.

The Fourier problems at the end of the Fourier chapter were written for this course by the author.

The licence is Creative Commons Attribution 4.0 International (https://creativecommons.org/licenses/by/4.0/). Changes: figures have been redrawn, and the wording of some problems has been lightly edited (for example "as shown below" → "as shown"), and the brief answers are taken from the answer keys provided by OpenStax. The original problems, figures and answers are available at the sources above. These notes are not endorsed by OpenStax.