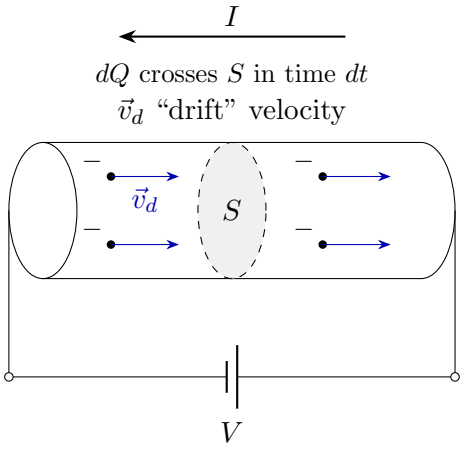
Chapter 2

Current, Resistance & DC Circuits

2.1 Current and Ohm's law



Current: let dQ be the charge that passes through the surface S (the wire's cross-section) in a time dt. Then

$$\boxed{I := \frac{dQ}{dt}}$$

measured in amperes, A = C/s (coulombs per second). A voltage source attached across the wire drives the current.

Ohm's law

Inside a conductor, the electric field drives a current. The local **current density** $\vec{J}$ (current per unit area, in A/m^2) is proportional to the field, and the factor of proportionality is the **resistivity** ρ, a property of the material:

$$\vec{E} = \rho\, \vec{J}.$$

Now consider a uniform wire of length L and cross-sectional area A_S (the area of S), with a voltage V across its ends. The field is uniform, so $E = V/L$, and the current density is uniform, so $I = J\, A_S$. Then

$$V = E\, L = \rho\, J\, L = \rho\, \frac{I}{A_S}\, L,$$

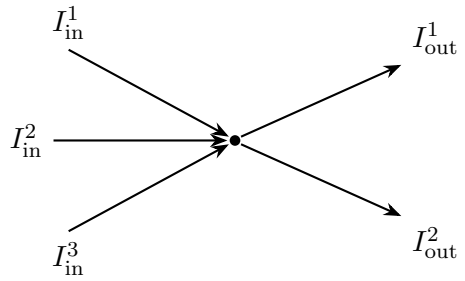
that is,

$$\boxed{V = R\, I}, \qquad \boxed{R = \frac{\rho L}{A_S}}. \tag{2.1}$$

This is **Ohm's law**: the voltage across the wire is proportional to the current through it, where the **resistance** R is a constant that depends on the electrical (through ρ) and geometric (L, A_S) properties of the wire. Resistance is measured in ohms, Ω = V/A.

2.2 Kirchhoff's rules

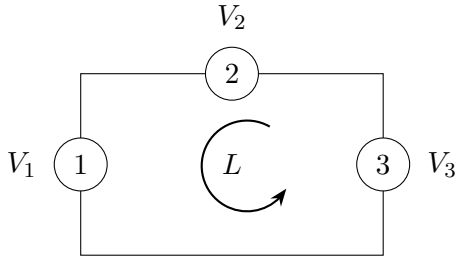
1. Junction rule



$$\sum_{i=1}^{N} I^i_{\text{in}} = \sum_{i=1}^{M} I^i_{\text{out}}$$

The total current into a junction equals the total current out. This is *conservation of charge*.

2. Loop rule

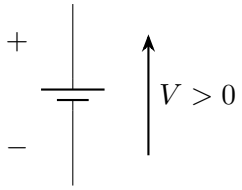


$$\sum_{\text{closed loop}} V_i = 0$$

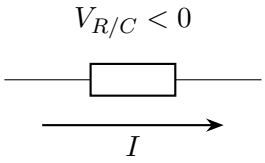
The voltages around any closed loop sum to zero. This is *conservation of energy*.

To apply the loop rule:

- Choose a loop, with a direction.
- Use sign conventions, e.g.:



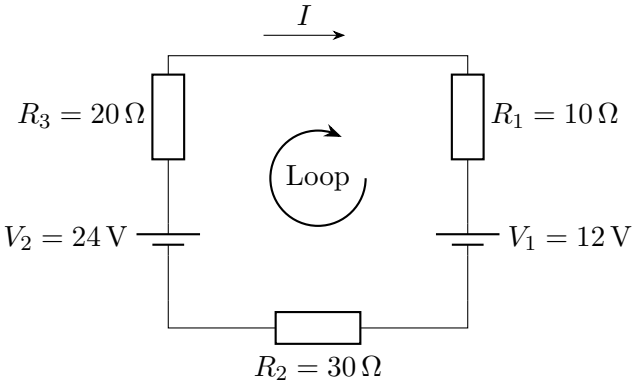
A battery contributes the voltage it induces, $V > 0$ (traversed from its $-$ to its $+$ terminal).



A resistor or capacitor (traversed along the current I) gives a voltage drop over the component, $V_{R/C} < 0$.

Example (OpenStax Example 10.7)

Find I.

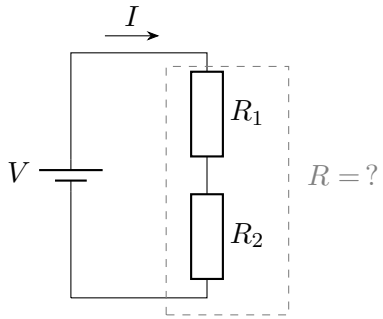


The loop rule (watch the signs!) gives

$$V_2 - IR_3 - IR_1 - V_1 - IR_2 = 0 \implies \boxed{I = \frac{V_2 - V_1}{R_3 + R_2 + R_1} = \frac{12\,\text{V}}{60\,\Omega} = 0.2\,\text{A}}$$

2.3 Resistors in series and parallel

Series

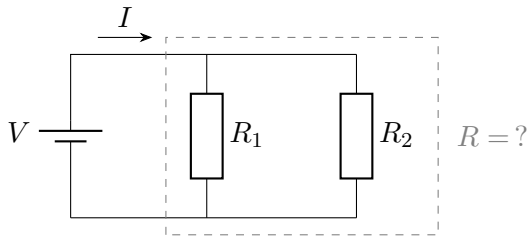


Let R be the effective resistance of the combination. By the junction rule, the same current flows through both resistors, $I_1 = I_2 = I$. By the loop rule, $V = V_1 + V_2$, so by (2.1)

$$R = \frac{V}{I} = \frac{R_1 I + R_2 I}{I}$$

$$\boxed{R = R_1 + R_2}$$

Parallel



Here the junction rule gives $I = I_1 + I_2$, and the loop rule gives equal voltages across the two resistors:

$$I = I_1 + I_2, \qquad V = V_1 = V_2.$$

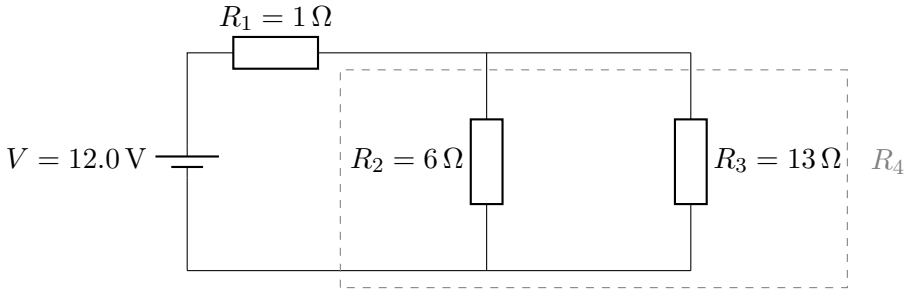
By Ohm's law,

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Example (OpenStax Example 10.4)

Find the equivalent resistance R of the circuit below.



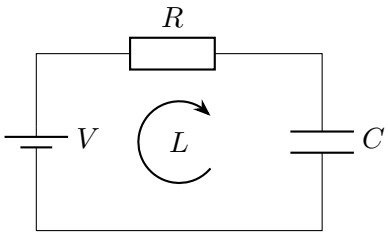
First find R_4, the combination of R_2 and R_3 in parallel:

$$\frac{1}{R_4} = \frac{1}{R_2} + \frac{1}{R_3} \implies R_4 = \frac{R_2 R_3}{R_2 + R_3} = \frac{78}{19}\,\Omega.$$

Then R_4 is in series with R_1:

$$\boxed{R = R_1 + R_4 = \left(1 + \frac{78}{19}\right)\Omega = 5.10\,\Omega}$$

2.4 RC circuits



Loop rule around the loop L:

$$V - V_R - V_C = 0,$$

with $V_R = IR$ (Ohm's law) and $V_C = Q/C$ (capacitance). Hence $V - IR - Q/C = 0$, and with $I = dQ/dt$,

$$\boxed{R\frac{dQ}{dt} + \frac{1}{C}Q = V(t)} \tag{2.2}$$

This is a differential equation for $Q(t)$.

Charging a capacitor

Take a constant applied voltage and no initial charge:

$$V(t) = V, \qquad Q(0) = 0.$$

Then (2.2) reads $RC\,\dfrac{dQ}{dt} = CV - Q$, i.e.

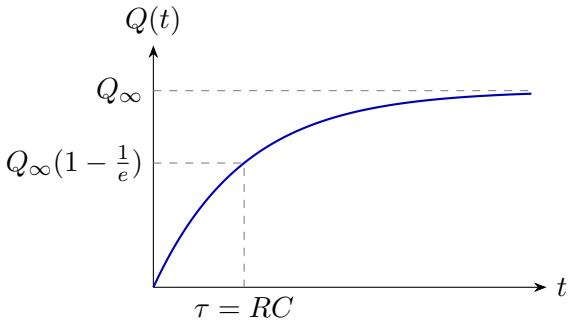
$$\frac{dQ}{CV - Q} = \frac{dt}{RC}.$$

Integrating, with the substitution $u = CV - Q$, $du = -dQ$:

$$\int_0^{Q(t)} \frac{dQ}{CV - Q} = \frac{t}{RC} \quad \Longrightarrow \quad -\int_{CV}^{CV-Q(t)} \frac{du}{u} = \ln\frac{CV}{CV - Q(t)} = \frac{t}{RC}.$$

Therefore $e^{t/RC} = \dfrac{CV}{CV - Q(t)}$, i.e.

$$\boxed{Q(t) = CV\left(1 - e^{-t/RC}\right)}$$



Call

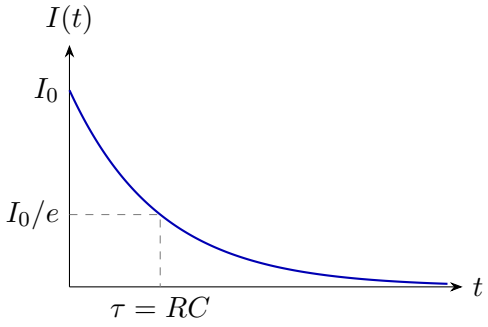
$$Q_\infty := CV, \qquad \tau := RC,$$

the final charge and the charging/decay time.

The current is

$$\boxed{I(t) = \frac{dQ}{dt} = \frac{CV}{RC}\,e^{-t/RC} = \frac{V}{R}\,e^{-t/RC}}$$

with $I_0 := \dfrac{V}{R}$.



Discharging a capacitor

Now remove the source and start from a charged capacitor:

$$V(t) = 0, \qquad Q(0) = CV_0.$$

Then (2.2) becomes

$$\frac{dQ}{dt} = -\frac{1}{RC}\,Q \quad \Longrightarrow \quad -\frac{dQ}{Q} = \frac{dt}{RC}. \tag{2.3}$$

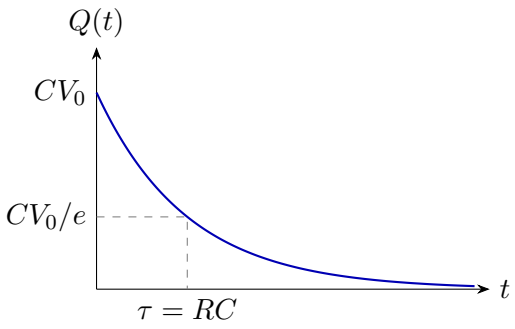
Integrating,

$$-\int_{CV_0}^{Q(t)} \frac{dQ}{Q} = \frac{t}{RC} \quad \Longrightarrow \quad \ln\frac{Q(t)}{CV_0} = -\frac{t}{RC},$$

so

$$\boxed{Q(t) = CV_0\, e^{-t/RC}}$$

$$\boxed{I(t) = \frac{dQ}{dt} = -\frac{V_0}{R}\, e^{-t/RC}}$$



Problems

Problems are taken from the sources listed under Sources and Attribution. Numbers refer to the end-of-chapter problems; figures have been redrawn.

OpenStax University Physics Vol. 2, Chapter 9: Current and Resistance

9.27 The charge on a capacitor in a circuit is modeled as $Q(t) = Q_{\max}\cos(\omega t + \phi)$. What is the current through the circuit as a function of time?

[Answer →]

9.37 What is the resistance of a 20.0 m-long piece of 12-gauge copper wire having a 2.053 mm diameter?

[Answer →]

9.51 A resistor rated at 250 kΩ is connected across two D cell batteries (each 1.50 V) in series, with a total voltage of 3.00 V. The manufacturer advertises that their resistors are within 5% of the rated value. What are the possible minimum current and maximum current through the resistor?

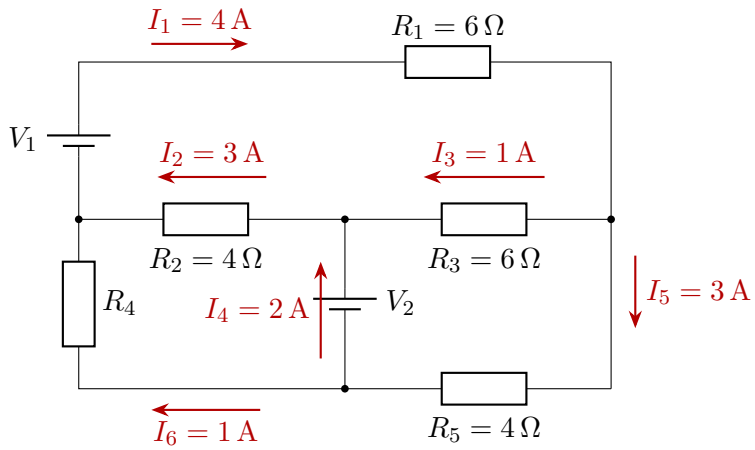
[Answer →]

OpenStax University Physics Vol. 2, Chapter 10: Direct-Current Circuits

10.31 Referring to the example combining series and parallel circuits in this lecture (Example 10.4 above), calculate I_3 in the following two different ways: (a) from the known values of I and I_2; (b) using Ohm's law for R_3. In both parts, explicitly show how you follow the steps in the problem-solving strategy for series and parallel resistors.

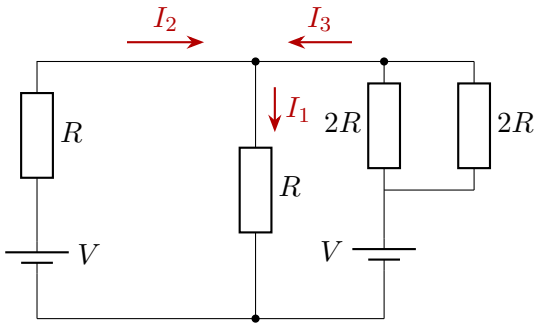
[Answer →]

10.39 Consider the circuit shown. Find V_1, V_2, and R_4.



[Answer →]

10.43 Consider the circuit shown. Write equations for the three currents in terms of R and V.



[Answer →]

10.53 A $500\,\Omega$ resistor, an uncharged $1.50\,\mu\text{F}$ capacitor, and a $6.16\,\text{V}$ emf are connected in series. (a) What is the initial current? (b) What is the RC time constant? (c) What is the current after one time constant? (d) What is the voltage on the capacitor after one time constant?

[Answer →]

10.57 If you wish to take a picture of a bullet traveling at $500\,\text{m/s}$, then a very brief flash of light produced by an RC discharge through a flash tube can limit blurring. Assuming $1.00\,\text{mm}$ of motion during one RC constant is acceptable, and given that the flash is driven by a $600\,\mu\text{F}$ capacitor, what is the resistance in the flash tube?

[Answer →]

Answers

Chapter 2: Current, Resistance & DC Circuits

9.27 $I(t) = -I_{\max}\sin(\omega t + \phi)$ [← Problem]

9.37 $R = 0.10\,\Omega$ [← Problem]

9.51 $R_{\min} = 2.375 \times 10^5\,\Omega,\ I_{\min} = 12.63\,\mu\text{A};\quad R_{\max} = 2.625 \times 10^5\,\Omega,\ I_{\max} = 11.43\,\mu\text{A}$ [← Problem]

10.31 (a) 0.74 A; (b) 0.742 A [← Problem]

10.39 $V_1 = 42\,\text{V},\ V_2 = 6\,\text{V},\ R_4 = 6\,\Omega$ [← Problem]

10.43 $I_1 = \frac{2}{3}\frac{V}{R},\ I_2 = \frac{V}{3R},\ I_3 = \frac{V}{3R}$ [← Problem]

10.53 (a) 12.3 mA; (b) 7.50×10^{-4} s; (c) 4.53 mA; (d) 3.89 V [← Problem]

10.57 $R = 3.33 \times 10^{-3}\,\Omega$ [← Problem]

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- OpenStax, *Calculus Volume 3*, by Gilbert Strang, Edwin "Jed" Herman et al. Licensed under CC BY 4.0. https://openstax.org/details/books/calculus-volume-3

The signal-processing chapter also draws on the following book, which is not openly licensed and is cited rather than reproduced: Marina Bosi and Richard E. Goldberg, *Introduction to Digital Audio Coding and Standards*, Kluwer Academic Publishers, 2003. The approximation of the threshold in quiet used there is taken from that book; the plot of it is our own.

The Fourier problems at the end of the Fourier chapter were written for this course by the author.

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